

DETERMINATION OF A GROUP OF THERMOPHYSICAL PROPERTIES

BY A DIFFERENTIAL-BRIDGE THERMOMETRIC METHOD

O. A. Gerashchenko, T. G. Grishchenko, L. V. Dekusha,
V. P. Salo, and A. V. Lapii

UDC 536.2.083

The theoretical basis is provided for the thermometric method of determining a group of thermophysical properties in the quasi-steady-state regime.

The solution of the one-dimensional heat-conduction problem for an infinite homogeneous plate of thickness $2H$ lies at the basis of the determination of the thermophysical properties (TPPs). The rate of change of the temperature at the boundaries of the plate is specified to be uniform and constant ($b = \text{const}$). In this case, naturally, the difference between the temperatures of the boundaries should also be constant. Here the boundary conditions are formulated as follows:

$$T(x, \tau = 0) = T_{0_2} + 0,5(T_{0_1} - T_{0_2}) \left(1 + \frac{x}{H}\right), \quad (1)$$

$$T(x = H, \tau) = T_{0_1} + b\tau, \quad T(x = -H, \tau) = T_{0_2} + b\tau.$$

In order to solve this problem, use is made of the method of superposition of the solutions of two subsidiary problems with the following boundary conditions: for the first subsidiary problem:

$$T(x, \tau = 0) = T_{0_2}, \quad T(x = H, \tau) = T_{0_2} + b\tau, \quad T(x = -H, \tau) = T_{0_2} + b\tau,$$

and for the second subsidiary problem,

$$T(x, \tau = 0) = -\frac{q_0 H}{\lambda} \left(1 + \frac{x}{H}\right),$$

$$-\lambda \frac{\partial T(x = H, \tau)}{\partial x} = q_0 = \text{const}, \quad T(x = -H, \tau) = 0.$$

The solutions of each of these subsidiary problems with respect to the temperature and temperature gradient are known [1, 2]. Their superposition leads to expressions for the required functions for the temperature and temperature gradient respectively of the main problem:

$$T(x, \tau) = \frac{T_{0_1} + T_{0_2}}{2} + \frac{T_{0_1} - T_{0_2}}{2} X + \frac{bH^2}{a} \left(Fo - \frac{1}{2} + \frac{X^2}{2}\right) +$$

$$+ \frac{bH^2}{a} \sum_{n=1}^{\infty} \frac{A_n}{\mu_n} \cos(\mu_n X) \exp(-\mu_n^2 Fo), \quad (2)$$

$$-\frac{\partial T(x, \tau)}{\partial x} = \frac{q_0}{\lambda} + \frac{bH}{a} X - \frac{bH}{a} \sum_{n=1}^{\infty} A_n \sin(\mu_n X) \exp(-\mu_n^2 Fo), \quad (3)$$

where $A_n = (-1)^{n+1} 2/\mu_n^2$, $\mu_n = (2n-1)\pi/2$.

The nature of the changes of the functions (2) and (3) with time at various cross sections over the thickness of the plate are shown in Fig. 1.

From an analysis of the exact solutions (2) and (3) of the main problem which have been obtained, it can be concluded that with increase of time the contribution of the terms with

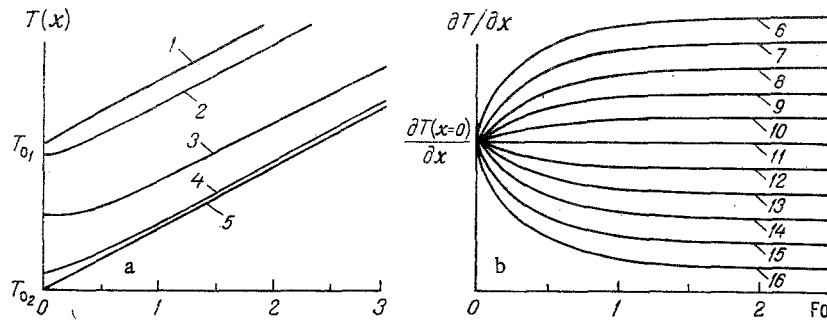


Fig. 1. Nature of the changes of temperature (a) and of temperature gradient (b) with respect to time for various cross sections over the thickness of the plate. 1-5) Graphs of the function $T = f(Fo)$ for $x/H = 1, 0.8, 0, -0.8, -1$, respectively; 6-16) Graphs of the function $\partial T/\partial x = f(Fo)$ for $x/H = 1, 0.8, 0.6, 0.4, 0.2, 0, -0.2, -0.4, -0.6, -0.8, -1$, respectively.

the sums of the series decreases, and for times which correspond to values of the Fourier number $Fo \geq 2$, they can be neglected, as can be seen from Fig. 1. Beginning at this time, a quasi-steady-state thermal regime is established in the plate under the boundary conditions (1), and the functions (2) and (3) for any cross section of the plate can be approximated by the following expressions with an error not exceeding 0.25%:

$$T(x, \tau) = \frac{T_{01} + T_{02}}{2} + \frac{bH^2}{a} \left(Fo - \frac{1}{2} \right) + \frac{T_{01} - T_{02}}{2} X + \frac{bH^2}{2a} X^2, \quad (4)$$

$$-\frac{\partial T(x, \tau)}{\partial x} = \frac{q_0}{\lambda} + \frac{bH}{a} X. \quad (5)$$

Figure 2 shows the nature of the change of the functions for the temperature (4) and for the temperature gradient (5) at a value $Fo = 2$ as a function of the cross section X , the rate of change of the temperature b , and the initial specified temperature difference $T_{01} - T_{02}$ between the boundaries of the plate.

From the expressions (4) and (5) and taking into account that $q_0 = 0.5\lambda (T_{01} - T_{02})/H$, a relationship is obtained for the temperature difference between two arbitrary cross sections x_1 and x_2 over the thickness of the plate:

$$T_1 - T_2 = T_1(x_1) - T_2(x_2) = \frac{H(X_1 - X_2)}{2\lambda} (q_1 + q_2), \quad (6)$$

which gives a formula for determining the thermal conductivity:

$$\lambda = 0.5H(X_1 - X_2)(q_1 + q_2)/(T_1 - T_2). \quad (7)$$

A formula for the volumetric heat capacity which is obtained by analogous means from the expression for the difference of the heat flux densities through the cross sections being considered has the following form:

$$c\rho = (q_1 - q_2)/(bH(X_1 - X_2)). \quad (8)$$

The known relationships interrelating all three thermophysical characteristics make it possible when (7) and (8) are known to obtain a formula for determining the temperature conductivity:

$$a = \frac{0.5H^2(X_1 - X_2)^2(q_1 + q_2)}{(T_1 - T_2)(q_1 - q_2)}. \quad (9)$$

The results obtained with respect to the temperature make it possible to obtain a volume-average temperature $\bar{T}(x)$ defined by the integral

$$T_{\text{ref}} = \bar{T}(x) = \frac{1}{H(X_1 - X_2)} \int_{x_2}^{x_1} T(x) dx,$$

in which the function $T(x)$ has the form of Eq. (4).

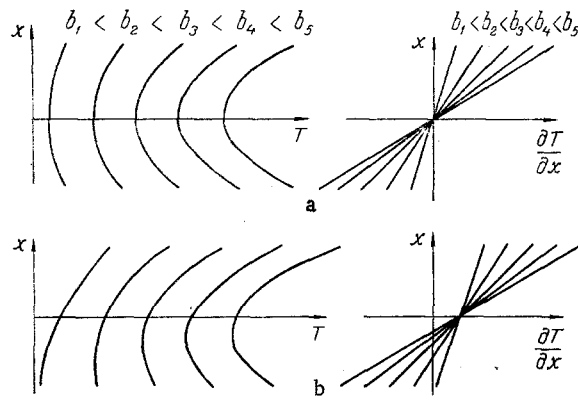


Fig. 2. Nature of the changes of the temperature and temperature gradient for various rates of change of the temperature at $Fo \geq 2$ and $T_{0_1} - T_{0_2} = 0$ (a) and $T_{0_1} - T_{0_2} = 25$ (b).

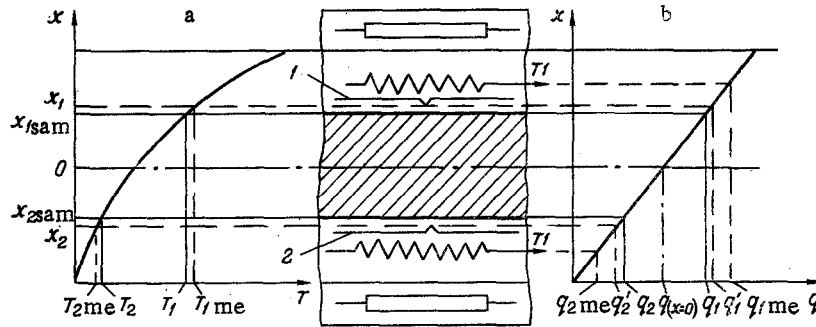


Fig. 3. Sketch of the device and of the distributions of the temperature (a) and of the heat flux density (b) over the thickness of the device.

The formula for calculating the reference temperature which is obtained after a number of rearrangements has the form:

$$T_{ref} = T_2 + \frac{T_1 - T_2}{3} \left(1 + \frac{q_2}{q_1 + q_2} \right).$$

The results obtained above for a single-layered plate can be applied to multilayered plates by taking into account the differences in the thermophysical properties of the various layers through their reduction to equivalent thicknesses, as recommended in [2].

In carrying out the recommended procedure the sample of the material to be investigated is placed between two blocks with programmable linear temperature changes with a specified constant temperature difference between the surfaces of the sample. Devices for measuring the temperatures and heat fluxes are placed between the blocks and the surfaces of the sample. A sketch of the device and of the distributions of the temperature and heat flux density over the thickness of the sample in the quasi-steady-state regime are shown in Fig. 3. Thermocouples can be used, for instance, as the devices for measuring the temperatures, and thermoelectric heat-measuring devices based on the auxiliary wall principle for measuring the heat flux densities [3].

By analyzing the accuracy potential of the measurement methods being considered the following conclusions can be drawn.

1. The sum of the heat fluxes appearing in Eq. (7) can be determined from the readings of two heat measuring devices placed at equal distances from the surfaces of the sample without error since the sum of the heat fluxes through any pair of cross sections equidistant from the central cross section of the sample ($x = 0$) is the same, i.e.,

$$q_1 + q_2 = q_{1sam} + q_{2me} = idem. \quad (10)$$

2. The temperature difference between the surfaces of the sample which appears in Eq.

(7) is not equal to the temperature difference measured by the thermocouples, since the coordinates of the thermocouples junctions (x_i) and of the adjacent surfaces of the sample ($x_{i\text{ sam}}$) do not exactly coincide, as seen from Fig. 3, i.e., $x_{i\text{ sam}} \neq x_i$ at $i = 1$ and 2 . This is caused by the presence of an interlayer between them which has a thermal resistance, as a result of which the temperatures measured by the thermocouples are not equal to the true temperatures of the samples surfaces, i.e., $T_{i\text{ me}} - T_i \neq 0$ for $i = 1$ and 2 (see Fig. 3a).

Thus, in order to ensure the correct determination of the thermal conductivity it is necessary to introduce a temperature correction δT which takes into account the thermal resistances of the ballast interlayers which have been referred to, as a result of which Eq. (7) is converted to the form

$$\lambda = \frac{0,5H(X_{1\text{ sam}} - X_{2\text{ sam}})(q_1 + q_2)}{(T_{1\text{ me}} - T_{2\text{ me}}) - \delta T}.$$

By introducing the equivalent thicknesses in (6), taking into account the multilayered nature of the system being considered, it is found that

$$\delta T = \frac{H(X_1 - X_{1\text{ sam}})_{\text{equ}}}{\lambda} \frac{q'_1 + q_1}{2} + \frac{H(X_{2\text{ sam}} - X_2)_{\text{equ}}}{\lambda} \frac{q_2 + q'_2}{2}, \quad (11)$$

where q'_1 and q'_2 are the heat flux densities at the cross-sections x_1 and x_2 ; q_1 and q_2 are the corresponding values at the cross sections $x_{1\text{ sam}}$ and $x_{2\text{ sam}}$.

If the expressions $H(|X_i - X_{i\text{ sam}}|)_{\text{equ}}/\lambda$ are denoted by R_i ($i = 1, 2$), each of which takes into account the thermal resistance of an interlayer between the section at which the thermocouple junction is fixed and the corresponding sample surface, then the temperature correction (11) can be written in terms of a sum of the products of the thermal resistances of these interlayers and half the sum of the heat flux densities through the cross sections bounding the interlayers, i.e.,

$$\delta T = R_1 \frac{q'_1 + q_1}{2} + R_2 \frac{q_2 + q'_2}{2}.$$

Under conditions in which the junctions of both thermocouples are placed identically and are at equal distances from the respective sample surfaces, it can be assumed that $R_1 = R_2 = R$, and also taking into account condition (10) for the linearity of measuring the heat flux densities in the quasi-steady-state regime, the temperature correction can be expressed as follows:

$$\delta T = R \frac{q'_1 + q'_2}{2} = R \frac{q_1 + q_2}{2},$$

and the calculation formula (7) becomes

$$\lambda = \frac{0,5H(X_{1\text{ sam}} - X_{2\text{ sam}})(q_1 + q_2)}{(T_{1\text{ me}} - T_{2\text{ me}}) - R(q_1 + q_2)}.$$

3. The difference in the heat flux densities passing through the surfaces of the sample which appears in Eq. (8) is not equal to the difference of the heat flux densities measured by means of the heat flow meters, which have finite thicknesses, as can be seen clearly from Fig. 3b. This comes about because the heat flux measured under unsteady-state conditions by means of a heat flow meter is proportional to the integral mean flux passing through the surfaces of the heat flow meter. Under quasi-steady-state conditions, this heat flux is proportional to the integral mean flux passing through the surfaces of the heat flow meter. Under quasi-steady-state conditions, this heat flux is proportional to the arithmetic mean of the heat fluxes entering and leaving the heat flow meter, i.e., it is equal to the heat flux through the central cross section of the heat flow meter.

Thus, in order to ensure the correct determination of the heat capacity, it is necessary to introduce a heat flux correction in Eq. (8) which takes into account the heat capacities of the ballast layers interposed between the central cross sections of the heat flow meters and the corresponding sample surfaces (see Fig. 3). As a result of this, Eq. (8) is rearranged to the form

$$c\rho = \frac{(q_{1\text{ me}} - q_{2\text{ me}}) - \delta q}{bH(X_{1\text{ sam}} - X_{2\text{ sam}})},$$

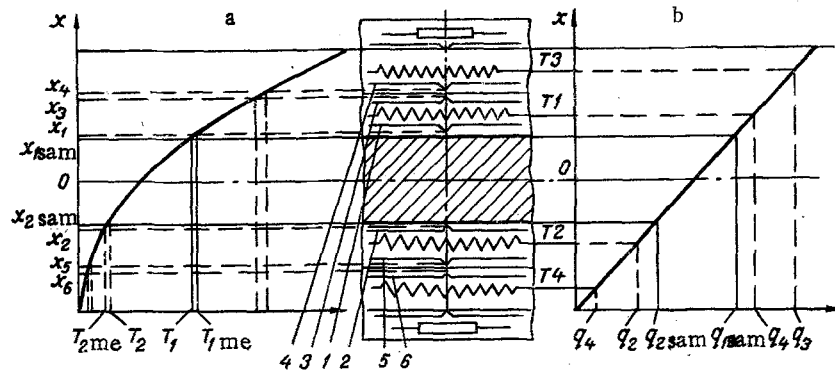


Fig. 4. Sketch of device with additionally introduced measuring devices and distributions of the temperature (a) and of the heat flux density (b).

where the heat flux correction δq can be expressed as follows:

$$\delta q = (q_{1me} - q_1) + (q_2 - q_{2me}) = bc\rho H [(X_1 - X_{1sam})_{equ} + (X_{2sam} - X_2)_{equ}].$$

Each of the products $C_i = c\rho H |X_i - X_{i\text{sam}}|_{equ}$ takes into account the volumetric heat capacity of one respective ballast layer ($i = 1, 2$) reduced to an equivalent thickness. Under conditions in which the heat flow meters are made identical to each other and are placed symmetrically relative to the middle cross section of the sample, it can be assumed that $C_1 = C_2 = C$, and the heat flux correction can then be expressed as follows:

$$\delta q = b(C_1 + C_2) = bC_2,$$

and the calculation formula (8) reduces to the form

$$c\rho = \frac{(q_{1me} - q_{2me}) - \delta q}{bH(X_{1sam} - X_{2sam})} = \frac{1}{H(X_{1sam} - X_{2sam})} \times \left(\frac{q_{1me} - q_{2me}}{b} - C_2 \right).$$

Thus, the validity of the determination of the thermophysical properties depends on the precision of the determination of the temperature correction δT and the heat flux correction δq . When operating over a wide range of temperatures it is difficult to take these corrections into account correctly because of their dependence on the temperature.

In order to improve the accuracy a differential method has been developed for correcting the measured temperatures and heat flux densities. This consists of introducing additional measuring elements into the equipment, the readings of which are proportional to the change in the temperature and heat flux which arises because of the presence of the ballast thermal resistance or heat capacity. A sketch of the device with the additional heat flow meters and thermocouples which need to be introduced is shown in Fig. 4, together with the temperature and heat flux distributions over the thickness of the system for the quasi-steady-state stage.

The correction of the measured temperature can be accomplished by using two pairs of differentially connected thermocouples, 3 and 4, and 5 and 6, respectively. The junctions of the thermocouples 3 and 5, 4 and 6 are placed in pairs at the same distances from the respective surfaces of the sample. In each differential thermocouple the junctions are separated by simulated interlayers possessing thermal resistances of R_{4-3} and R_{5-6} , respectively, so that

$$\frac{R_1}{R_{3-4}} = \frac{R_2}{R_{5-6}} = \frac{\delta T}{\Delta T_{4-3} + \Delta T_{5-6}} = \kappa.$$

When allowance is made for the relative temperature correction, the calculation formula (7) is transformed to

$$\lambda = \frac{0,5H(X_{1sam} - X_{2sam})(q_{1me} + q_{2me})}{\Delta T_{1-2me} - \kappa(\Delta T_{4-3} + \Delta T_{5-6})}. \quad (12)$$

Let us now return to the volumetric heat capacity. The quantity of heat accumulated by the ballast layers mentioned above (between the middle cross section of the heat flow

meters of the main pair form check elements for the interlayers between the central cross sections of the contacting heat flow meters. In this case, the sum of the differences of the heat fluxes measured respectively by the pairs of heat flow meters T1 and T3, T2 and T4, is proportional to the quantity of heat accumulated by the check element. Here it is also obvious that

$$\frac{\delta q}{\Delta q_{3-1} + \Delta q_{2-4}} = \frac{C_1}{C_\Sigma} = \gamma.$$

When allowance is made for the heat flux correction the calculation formula (8) for the volumetric heat capacity can be written as

$$c\rho = \frac{1}{bH} [(q_{1me} - q_{2me}) - \gamma(\Delta q_{3-1} + \Delta q_{2-4})]. \quad (13)$$

When the conditions of identical placement of the thermocouple junctions and similarity of the thicknesses of the simulated interlayers (i.e., the interlayers between the junctions in both differential thermocouples 3-4 and 5-6) are satisfied, and also the heat flow meters are placed symmetrically and the measurements are made under the conditions $T_{0_1} - T_{0_2} \leq 10$ K, then both of the relative corrections κ and γ are independent of the temperature. Their values can be determined experimentally in a "blank run" test, i.e., in the absence of a sample between the heating plates, which corresponds to the condition $X_{1\text{ sam}} - X_{2\text{ sam}} = 0$.

By introducing the relative temperature and heat flux corrections, the temperature and heat flux differences measured on the surfaces of the sample are completely corrected. However, an important drawback of the quasi-steady-state method remains, namely, the need to measure the rate of change of the temperature, which can be accompanied by considerable error. The presence of two pairs of thermocouples makes it possible to avoid the determination of the rate of change of the temperature in the course of the experiment, and to use instead the results of the measurements of the changes of the heat flux density in the check elements [4].

The following system of equations can be written for a single-layer plate (see Fig. 4):

$$q_3 - q_1 = C_{3-1}b, \quad q_1 - q_2 = (C_{1-2} + C_{\text{sam}})b, \quad q_2 - q_4 = C_{2-4}b, \quad (14)$$

where $C_{\text{sam}} = c\rho H(X_{1\text{ sam}} - X_{2\text{ sam}})$ is the volumetric heat capacity of the sample: $C_{1-2} = C_\Sigma$ is the summed volumetric heat capacity of the ballast layers which occur between the surfaces of the sample and the central cross sections of the heat flow meters adjacent to them; C_{3-1} and C_{2-4} are the effective volumetric heat capacities of the layers occurring between the central cross sections of the heat flow meters T1 and T3, T2 and T4, respectively, which are in pairwise contact with each other, i.e., the check elements.

The solution of the system of equations (14) gives an expression for the rate of change of the temperature:

$$b = \frac{(q_3 - q_1) + (q_2 - q_4)}{C_{3-1} + C_{2-4}} = \frac{\Delta q_{3-1} + \Delta q_{2-4}}{C'_\Sigma},$$

and substitution of this into Eq. (12) leads to the following calculation equation for the volumetric heat capacity of the sample:

$$c\rho = \frac{C'_\Sigma}{H(X_{1\text{ sam}} - X_{2\text{ sam}})} \left(\frac{q_{1me} - q_{2me}}{\Delta q_{3-1} + \Delta q_{2-4}} - \gamma \right). \quad (15)$$

By using the calculation equations after correction (Eqs. (12) and (15)), the calculation equation for the temperature conductivity can be rearranged to

$$a = \frac{0,5H^2(X_{1\text{ sam}} - X_{2\text{ sam}})^2(q_{1me} + q_{2me})}{[\Delta T_{1-2\text{ me}} - \kappa(\Delta T_{4-3} + \Delta T_{5-6})]C'_\Sigma \left(\frac{q_{1me} - q_{2me}}{\Delta q_{3-1} + \Delta q_{2-4}} - \gamma \right)}. \quad (16)$$

The present method for determining the thermophysical properties is a differential method, on the one hand, since use is made of differences of the readings of the devices for measuring the heat fluxes. On the other hand, the characteristics to be measured (a and $c\rho$) are found from the ratios of the quantities of heat which are accumulated by the sample of the material being investigated and by the check elements, the thermophysical properties of

which are known, i.e., by a bridge method [5]. Thus, a new method has been developed [4] for determining the group of thermophysical properties which can be classified as a differential-bridge thermometric method, the calculation formulas for which are given by equations (12), (15), and (16).

NOTATION

x, τ , local values of the spatial coordinate and time; H , half-thickness of plate; T , temperature; T_{01}, T_{02} , initial constant temperatures of surfaces; b , rate of change of temperature; q , heat flux density; q_0 , initial heat flux density through surface of plate whose coordinate is $x = H$; q_1, q_2 , heat flux densities at cross sections $x = x_1, x = x_2$; λ , thermal conductivity; a , temperature conductivity; $X = x/H$, dimensionless coordinate; $Fo = a\tau/x^2$, Fourier number; $c\rho$, volumetric heat capacity, $c\rho = \lambda/a$; $\delta T, \delta q$, temperature and heat flux corrections; κ, γ , relative temperature and heat flux corrections; R , thermal resistance.

LITERATURE CITED

1. A. V. Lykov, Theory of Heat Conduction [in Russian], Moscow (1967).
2. A. I. Pekhovich and V. M. Zhidkikh, Calculation of the Thermal Conditions in Solids [in Russian], Leningrad (1976).
3. O. A. Gerashchenko, Fundamentals of Thermometry [in Russian], Kiev (1971).
4. T. G. Grishchenko, O. A. Gerashchenko, L. V. Dekusha, and N. A. Sintsov, Inventor's Certificate No. 1 165 957, Class G 01 N 25/18, "A means for determining a group of thermophysical properties of planar samples and equipment for its realization," Byull. Izobret., No. 25 (1986).
5. L. I. Anatyukh, Thermoelements and Thermoelectric Devices (handbook) [in Russian], Kiev (1979).

METHOD FOR DETERMINING THE THERMOPHYSICAL CHARACTERISTICS OF ORTHOTROPIC BODIES

Yu. M. Kolyano

UDC 536.2.02

The solution for the inverse coefficient problem of heat conduction for an orthotropic body is proposed.

There exists a nondestructive monitoring method for determining the coefficients of thermal conductivity of orthotropic bodies [1]. In the method a standard sample (isotropic half-space), whose thermal conductivity is known, is heated together with the sample of interest, placed in series, with a mobile point source of heat moved along the surface of the samples at a constant rate and the excess limiting temperature of the surface of the samples along the line of heating is measured with the help of a temperature sensor moved at the same rate as the source at a fixed distance from it. The sample under study is made with two mutually perpendicular flat surfaces, perpendicular to its principal axis of heat conduction, and scanning over the flat surfaces along each of the three principal axes of heat conduction is performed in sequence. The coefficient of thermal diffusivity and the volume heat capacity are not determined.

To determine the complex of thermophysical characteristics of orthotropic bodies by the method of nondestructive monitoring, we shall examine three samples in the form of orthotropic half-spaces $z \geq 0, x \geq 0, y \geq 0$, over whose surfaces $z = 0, x = 0$, and $y = 0$ a source of heat with power q [W] and a sensor for measuring the temperature at a fixed distance ℓ for the heat source (Fig. 1) move with a constant velocity v in the positive direction along

Institute of Applied Problems in Mechanics and Mathematics, Academy of Sciences of the Ukrainian SSR, Lvov. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 54, No. 4, pp. 596-600, April, 1988. Original article submitted November 10, 1986.